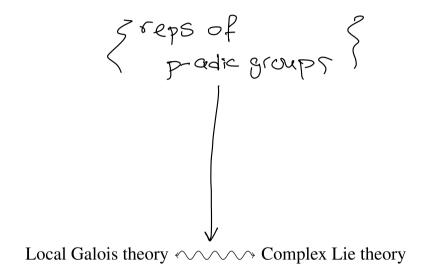
A brief tour of the Local Langlands Correspondence

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I. The local Langlands correspondence: my perspective

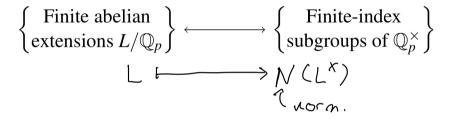
The local Langlands correspondence predicts:



Where does the LLC core from?

Local class field theory (Hasse, 1930)

There is an explicit bijection



such that

 $\operatorname{Gal}(L/\mathbb{Q}_p) \simeq \mathbb{Q}_p^{\times}/N(L^{\times}).$

What about noncepelian exts? From CFT to LLC \mathcal{W} the Weil group of \mathbb{Q}_p $\mathcal{W} \subseteq \mathcal{G}_{all}(\overline{\mathbb{Q}_{p}}/\mathbb{Q}_p)$ Sreps of $\zeta \longrightarrow Gal(L/Q_p)$ $\zeta \longrightarrow Gal(L/Q_p)$ $\zeta \longrightarrow Gal(L/Q_p) \longrightarrow C^*$

The LLC, more explicitly

G split, simple connected reductive group over $\mathbb{Q}_p \quad \text{ex } G = SL_n$, $\mathcal{P}GL_n$ $G = G(\mathbb{Q}_p)$ $SO_n \mid G_2 \mid E_8$

 $G^{\vee} \text{ dual group of } \mathbf{G} (\mathbb{C}\text{-points})$ $\underbrace{e_X} G = \operatorname{SL}_n(\mathbb{Q}_p) \implies G^{\vee} = \operatorname{PGL}_n(\mathbb{C})$ $G = \operatorname{SO}_{2n+1}(\mathbb{Q}_p) \implies G^{\vee} = \operatorname{Sp}_{2n}(\mathbb{C})$

The LLC predicts an explicit map

$$\begin{cases} \text{smooth/representations} \\ \text{of } G \end{cases} \longrightarrow \begin{cases} \text{Langlands parameters} \\ \varphi : \mathcal{W} \times \operatorname{SL}_2(\mathbb{C}) \to G^{\vee} \end{cases}$$

satisfying certain properties.

T. Depth Suppose P(W) is finite. => 9 determines a finite Galois ext Lp/QD. with $Gal(Lp/Q_p) \simeq P(W)$.

"windly ranified" Gal. grp is a p-group. (Lp)" "tanaly ranified" |> cyclic, order prime to p. |Lp)" "unramified" | cyclic

$$\begin{cases} \operatorname{smooth}^{\operatorname{irred}} \\ \left\{ \operatorname{smooth}^{\operatorname{irred}} \right\} \longrightarrow \begin{cases} \operatorname{Langlands parameters} \\ \varphi : \mathcal{W} \times \operatorname{SL}_{2}(\mathbb{C}) \to G^{\vee} \end{cases}$$

$$\underset{u \in \mathbb{N}}{\operatorname{supple}} \\ \left\{ \operatorname{unipplent}^{\operatorname{rep}} \right\} \xrightarrow{\operatorname{LLCun}} \\ \left\{ \operatorname{unipplent}^{\operatorname{rep}} \right\} \xrightarrow{\operatorname{LLCun}} \\ \left\{ \operatorname{unreaut}^{\operatorname{rep}} \right\} \xrightarrow{\operatorname{unreaut}^{\operatorname{rep}}} \\ \left\{ \operatorname{unreaut}^{\operatorname{unreaut}^{\operatorname{rep}}} \right\} \xrightarrow{\operatorname{unreaut}^{\operatorname{rep}}} \\ \left\{ \operatorname{unreaut}^{\operatorname{unreaut}^{\operatorname{rep}}} \\ \left\{ \operatorname{unreaut}^{\operatorname{unreaut}^{\operatorname{rep}}} \right\} \xrightarrow{\operatorname{unreaut}^{$$

What reps do these correspond to?

$$\underbrace{SL_2(\mathbb{Z}_p)}_{p \in rachoric^*} \geq \underbrace{\begin{pmatrix} 1+p\mathbb{Z}_p & p\mathbb{Z}_p \\ p\mathbb{Z}_p & 1+p\mathbb{Z}_p \end{pmatrix}}_{P_1} \geq \underbrace{\begin{pmatrix} 1+p^2\mathbb{Z}_p & p^2\mathbb{Z}_p \\ p^2\mathbb{Z}_p & 1+p^2\mathbb{Z}_p \end{pmatrix}}_{P_2} \geq \dots$$

Given a smooth irrep V of G, can use M-P filtrations to define depth of V. Idea: For which r is $V^{Pr} \neq 0$? ex V is depth zero if I a poschoric P s.f. $V^{P} \neq 0$.



What's known?
• all unipotent irreps have been constructed,
• A potential LLC un has been defined.
shill need to check desired properties of the LLC.
How?
$$P = SL_2(Z_p)$$
 $P/p_1 \simeq SL_2(IF_p)$
 P_1

 $\overline{1}$. Positive-depth representations If p>>0, then we know how to get all irreps of G. lf G is classical - and p= 2 But if p is small and G is exceptional, we don't.

